Fractal Energy: Combining fractal dimension of fracture surfaces and Size Specific Energy to describe comminution

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1 Abstract

The ubiquitous 'P80' is a useful descriptor of a particle size distribution, except when it isn't. First described by Fred Bond as the point where the linear segment of the size distribution starts, it is valid only when the feed and product size distributions are parallel in log/log space. Many alternative approaches have been developed to enable the continued use of P80 outside of these limitations, but as the number of exceptions grows, a re-think is required.

Although only coined recently, Size Specific Energy (SSE) has been used to describe comminution systems for over a century, since well before Bond. It is a tried-and-true methodology that assesses the energy consumed to generate fines. When assessed at a range of fine marker sizes, SSE can also describe the fractal dimension of the fracture surfaces generated. Fractal Energy combines SSE and the fractal dimension in an approach that has applicability to ore characterisation, circuit design and optimisation. This paper will present the value proposition for adopting this approach and present case studies for different ores, comminution equipment and testing apparatus.

2 Introduction

In its simplest form, comminution is the process of making big rocks smaller. Therefore, any estimate of breakage energy requires a description of the feed and product size distributions and the energy consumed to get from the feed to the product. The screen size at which 80% of the mass passes (P80) was used in Bond's third theory of comminution (1952) and is now the standard convention for describing a size distribution. However, the P80 (or any other single percent passing size) is only valid for predicting specific energy when the feed and product size distributions are parallel in log-log space. This assumption was valid for Bond' standard comminution circuit containing staged crushing, open-circuit rod milling and closed-circuit ball milling. However, modern comminution circuits containing autogenous (AG) mills, semi-autogenous (SAG) mills, high pressure grinding rolls (HPGR) and stirred media mills may not typically conform to this standard. Therefore, the description of particle size distributions needs to be updated to be applicable for modern comminution circuits.

Size Specific Energy (SSE) describes the energy required to generate new material finer than a specific marker size and can be used as an alternative measure of breakage energy requirements. This method relies on the Rittinger (1867) hypothesis that comminution energy is proportional to surface area generation, because the fines contain the large majority of surface area. In SAG and ball milling circuits, the majority of surface area is found in the -75 µm size fraction, this is the standard SSE marker size that has been used in previous investigations (Davis, 1919; Hukki and Allenius, 1968; Mokken, 1979; Schonert, 1988; Levin, 1999; Musa and Morrison, 2009; Hilden and Suthers, 2010; Powell, et al., 2010 and Ballantyne, et al., 2015). Although 75 µm (SSE75) has been shown to be useful as a marker size, Ballantyne (2019) showed that this is not always the case. Grinding to a P80 finer than 75 µm precludes the use of SSE75. However, for these finer grinds an effective SSE75 can be calculated from the SSE at a finer marker size and the gradient of the relationship between SSE and marker size.

Subsequent investigations have revealed that the relationship between SSE and marker size can be used to calculate the fractal dimension of the fracture surface area. Fractals are patterns that are found in mathematics and nature that repeat themselves and appear similar at various scales, such as leaves, snow-flakes, lightning and coastlines. The fundamental geometry of fractals can be described mathematically in terms of their fractal dimension. Turcotte (1986) and later Carpinteri and Pugno (2002) studied the fractal nature of rock fracture. They found that the fractal dimension is a fundamental indication of the degree to which energy dissipates in a volume (Kick, 1885) or on a surface (Rittinger, 1867). The fractal dimension may also provide a solution to the settlement between the three theories proposed by Hukki (1961). The fractal dimension of breakage also has the benefit of having a physical significance; low fractal dimensions relate to a breakage mechanism associated with bulk splitting, whereas high fractal dimensions relate to pulverisation (Turcotte,1986).

This paper presents the Fractal Energy approach which combines SSE and the fractal dimension of breakage to describe how comminution energy transforms particle size distributions.

3 Methodology

The Fractal Energy approach incorporates a combination of SSE and fractal dimension to allow for the complete characterisation of comminution energy efficiency across a full particle size distribution. Firstly, SSE is the measure of energy required to generate new material finer than a specific marker size. A 75 µm marker size is typically used for SAG and ball milling circuits and is denoted by SSE75. The calculation for SSE75 is included below:

$$SSE75 = \frac{mill\ pinion\ power\ (kW)}{throughput\ (t/h) \times (\% - 75\mu m_{prod} - \% - 75\mu m_{feed})}$$

SSE can be calculated from breakage data across all scales, laboratory to operations.

Assessing the SSE across a range of marker sizes allows the assessment of the fractal dimension of fracture surfaces. Figure 1 presents an example of the relationship between SSE and marker size for a typical SAG and ball milling circuit. A power law relationship between marker size and SSE creates a linear correlation in log/log space. This specific example shows that the SAG and ball mill achieved similar gradient and that the efficiency of the ball mill to the SAG mill was relative to the feed slurry density in the ball mill. A distinct upturn in the ball mill relationship occurs at coarser marker sizes when an inappropriately high marker size is chosen. This typically corresponds to markers sizes that are greater than the P80 of the product size distribution. The fractal dimension of breakage is simply calculated as 3+ the gradient of the linear component of the power law function.

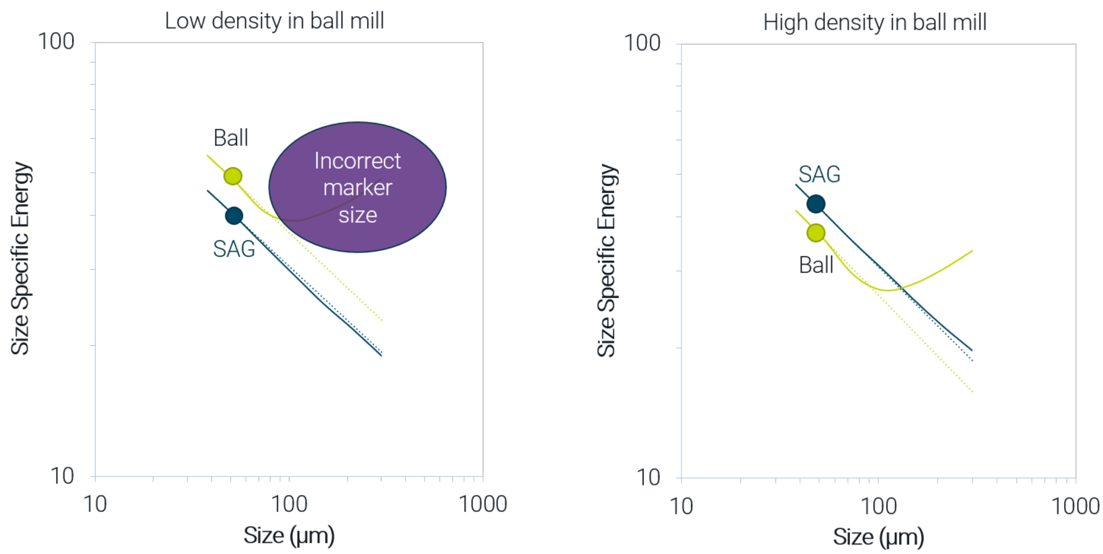

Figure 1 - Examples of using SSE across a range of marker sizes and the impact of ball mill charge density (after Ballantyne, 2019)

The gradient of the relationship in Figure 1 is -0.5 which corresponds to a fractal dimension of 2.5 (2.5 = -5 + 3). This is the fractal dimension that results when the feed and product size distributions have a slope of 0.5 in log/log space and corresponds to Bond's third law of breakage. Similarly, the predicted specific energy from the Fractal Energy approach is proportional to the Morrell approach when the fractal dimension of breakage equalled Morrell's (2008) exponent parameter (3 - 0.295 + P_{80}/1000000). Therefore, in addition to providing accurate predictions when the slope of the feed and product sizes were not similar, SSE may be applicable over a wider range of cases than Bond or Morrell.

4 Fractal dimension for different ores

The fractal dimension is an inherent breakage characteristic that is dependant on both the ore the breakage mechanism imparted from the comminution device and the efficiency of any close-circuit classifiers. Ballantyne and Giblett (2019) benchmarked fifteen of Newmont's operations using size specific energy, Morrell and Bond work index. The fractal dimension for each piece of equipment surveyed by Newmont was also assessed using the fractal energy approach. No significant difference was found between the different mill types (SAG and ball). However, the fractal dimensions observed for different ores were different, with non-overlapping

distributions (

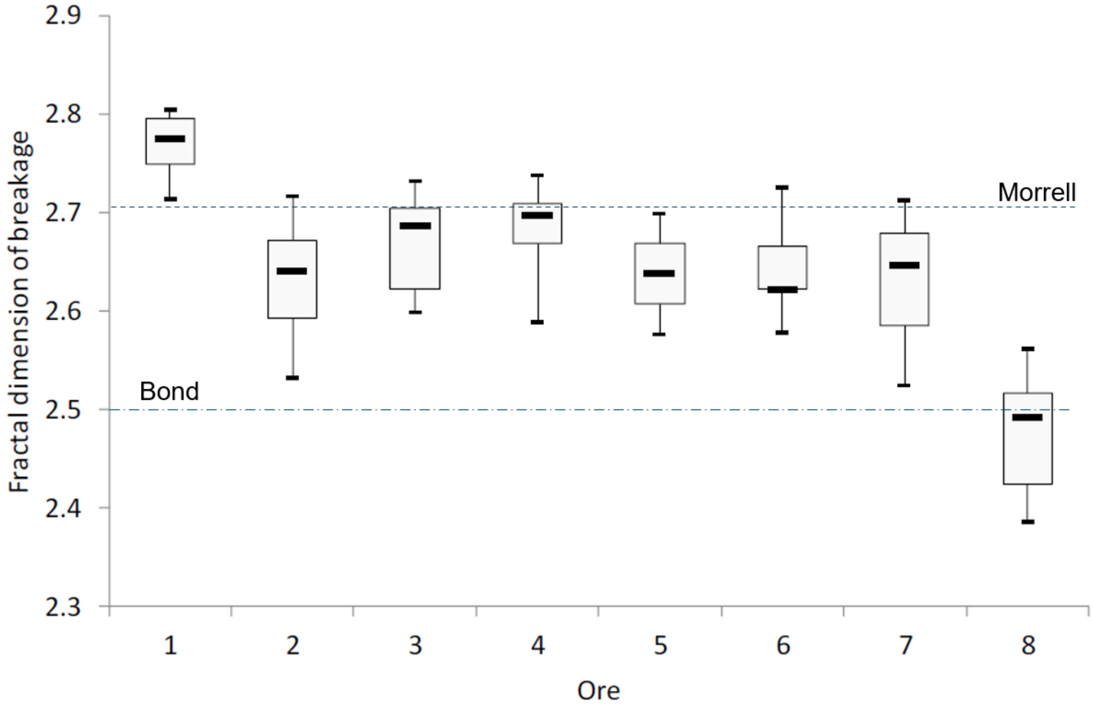

Figure 2). For instance, ore 1 produced fractal dimensions between 2.7 and 2.8 for both SAG and ball milling, whereas similar milling environments with ore-type 8 produced fractal dimensions between 2.4 and 2.55 (similar to Bond). The majority of the surveys resulted in a larger fractal dimension than Bond would predict (2.5) and more predominately in the range Morrell predicts. The difference in proportion of clays, mineral association and fracture frequency are all hypothesised to impact the fractal dimension.

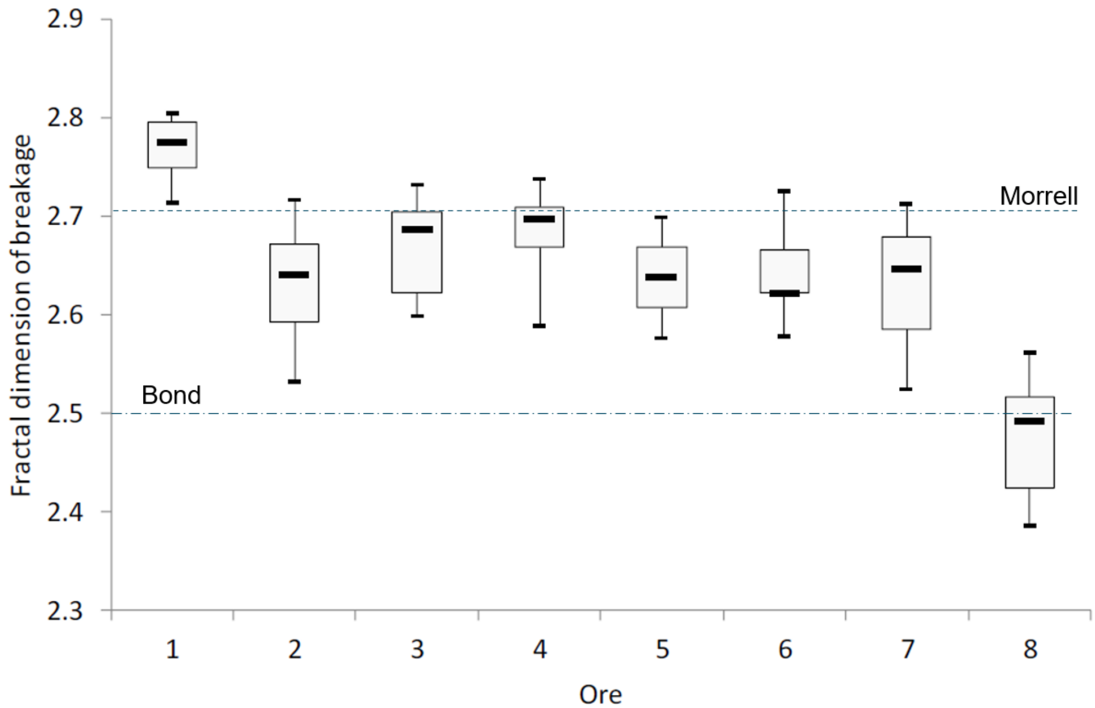

Figure 2 - Fractal dimension of breakage for 8 different ores displayed as a box-and-whisker plot highlighting the minimum, maximum, median and interquartile range. The Fractal dimension equivalent to Bond and Morrell are presented.

4.1 SSE and Fractal dimension for different comminution equipment

SSE75 and the fractal dimension can be an effective measure of the comminution energy efficiency of different equipment. The ratio of SSE75 for individual equipment and the whole circuit provides an effective relative energy efficiency measurement. Figure 3 shows that when SSE75 of the individual SAG and ball mills are compared to the whole circuit, the frequency distributions show that both mills are normally distributed around a median of approximately 100% with similar standard deviations. This means that the energy efficiency of both SAG and ball mills is similar when it is related to the generation of new -75 µm. The distribution shows that 50% of the mill achieve an SSE of +/- 10% of the total circuit, irrespective of whether they are SAG or ball mills.

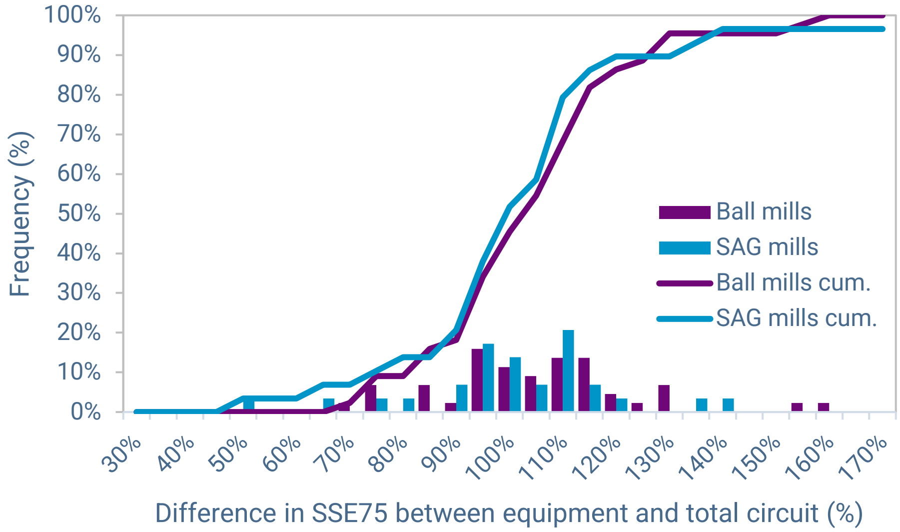

Figure 3 - Distribution of the ratio of SSE75 for SAG and ball mills in comparison to the total circuit.

A similar analysis was completed for the fractal dimension of breakage for SAG and ball mills in comparison to the whole circuit. Figure 4 shows the fractal dimension of breakage for the individual SAG and ball mills relative to the whole circuit. The frequency distributions show that both mills are normally distributed around a median of approximately 100% with similar standard deviations of only 2.6% and 3.8% for SAG and ball mills. The standard deviation of the entire database of fractal distributions shows a standard deviation of 16% showing that the variance between different ore-types is larger than the variance between SAG and ball mills.

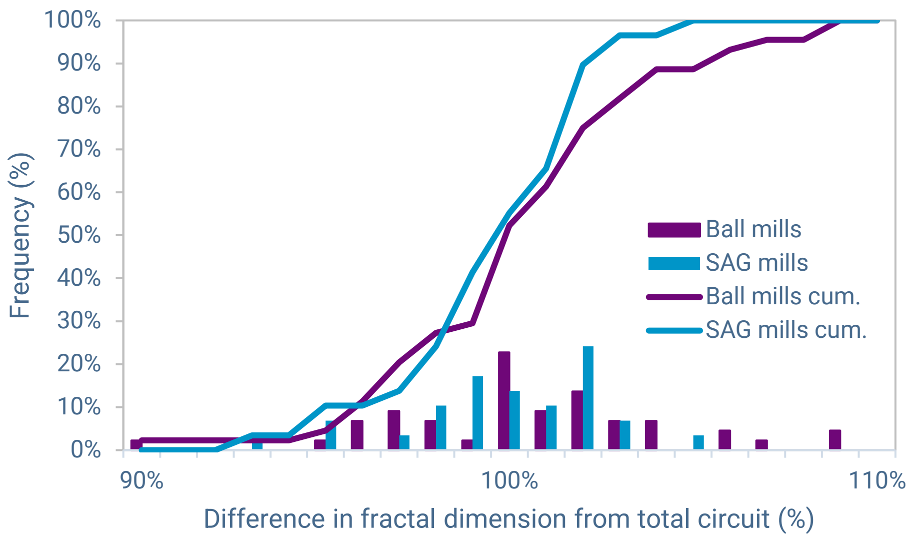

Figure 4 - Distributions of the ratio in fractal dimension of breakage for SAG and ball mills in comparison to the total circuit

Similar relationships for both SSE75 and fractal distributions have been seen for other mill types, but the scarcity of data means that these can't be analysed with the same confidence. Initial analysis of HPGR and stirred mills indicates that they tend to produce fractal dimensions of breakage that are significantly different. Figure 5 presents some preliminary analysis of the fractal dimension of breakage from HPGR and stirred mills. The fractal dimension of breakage in the HPGR tends to be higher than the stirred mills. Because of the packed-bed compression breakage mechanism, HPGRs tend to apply comminution energy over a wide range of feed particle sizes. This breakage mechanism results in increased fines production relative to P80, and a higher fractal dimension. On the other hand, stirred mills tend to have efficient internal and external classification that target the comminution energy on the coarsest size fraction and result in a sharper product size distribution and a lower fractal dimension.

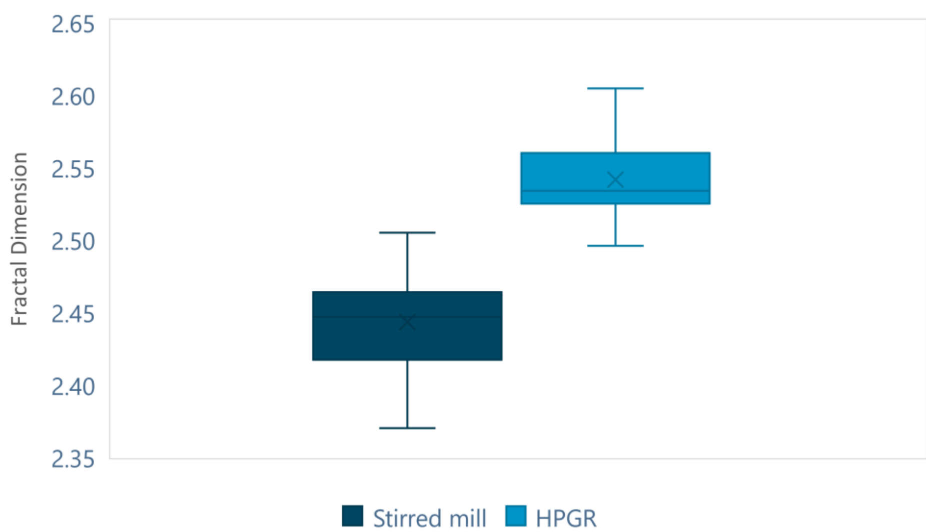

Figure 5 - Fractal dimension of breakage for stirred mills and HPGR

5 SSE and fractal dimension for laboratory ore characterisation

To assess the fractal energy as a function of ore characteristics, the comminution mechanism needs to be consistent. Results from the Bond ball mill work index test provide a relatively consistent basis on which to assess the range of fractal energy for different ores. The size specific energy was calculated by back calculating the specific energy assumptions inherent in the Bond ball mill work index calculation. Ballantyne, et. al. (2018) provides the full proof of the calculation, which is reproduced below:

$$\mathrm{SSE75(kWh/t{-}75\ \mu m)} = 49.1/[\mathrm{P}_1^{0.23}\mathrm{Gpr}^{0.82}(\%{-}75\ \mu\mathrm{m}_{\mathrm{prod}} - \%{-}75\ \mu\mathrm{m}_{\mathrm{feed}})].$$

Unfortunately, the method used for the Bond ball mill test means that the screening size fractions for the feed and product do not overlay. The finest screen size measured on the feed is the closing screen for the test, and the product is 100% passing this screen. Therefore, to calculate the SSE, the fines in the feed are extrapolated from the feed size distribution using linear interpolation in log/log space. In this way, the SSE at a range of marker sizes was calculated for a large database of Bond test results. The SSE75 was normally distributed with a median of 21.4 kWh/t-75 µm with an interquartile range of 17 to 26. The SSE75 averaged 1.45 times larger than the BWi, and approaches 1:1 equivalency for a marker size of 145 µm.

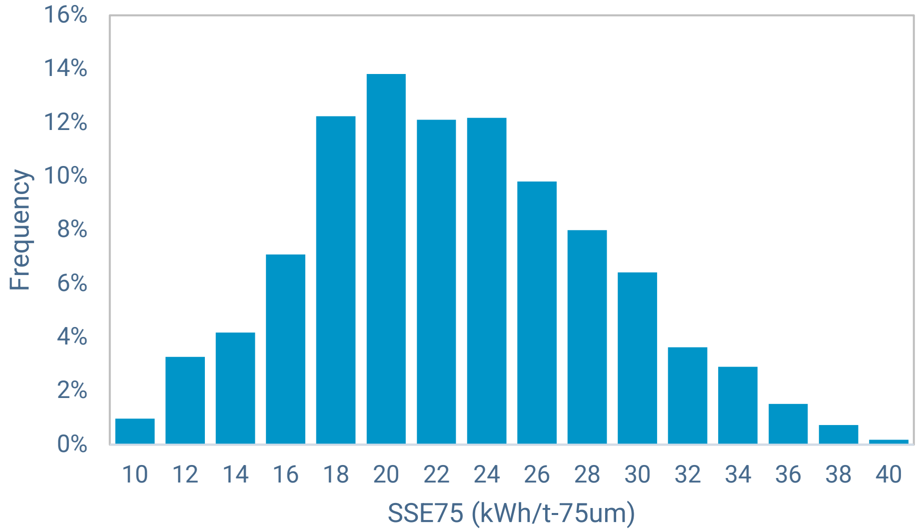

Figure 6 - Distribution of SSE75 from Bond ball mill work index tests

The power-law relationship between marker size and SSE was used to calculate the fractal dimension of breakage for the Bond ball mill tests. The gradient of the product size distribution was much more significant in determining the fractal dimension than the feed size distribution. In addition, no-relationship was found between either closing screen size or work index and the fractal dimension. The fractal dimension had a skewed distribution with 2.5 the most frequent result (Figure 7). A fractal dimension of 2.5 corresponds to Bond's third law of comminution. However, although this was the most frequent, the fractal dimension ranged from 1 to 3 for different tests. This shows that although the Bond relationship is correct in the majority of cases, it is not always the case, and the fractal energy approach provides a more accurate indication of breakage response.

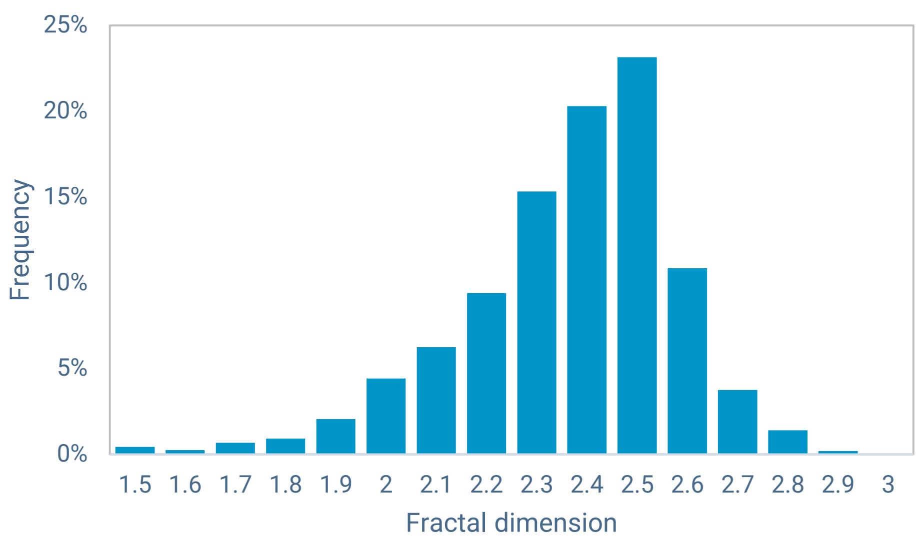

Figure 7 - Distribution of fractal dimension from Bond ball mill work index tests

A database of industrial surveys was also interrogated to determine the fractal dimension of breakage. Figure 8 presents the distribution of fractal dimensions for the industrial surveys and Bond ball mill tests as a box and whisker plot that shows the minimum, maximum, interquartile range, media and mean. The Bond ball mill test results tended to produce lower fractal dimensions with 75% of tests producing a fractal dimension below 2.5. On the other hand, the industrial surveys tended to produce fractal dimensions above 2.5, with only 25% of tests producing a fractal dimension below 2.5. Although the fundamental driver for this is unclear, it may be due to the difference in classification efficiency between screens (used in the Bond test) and hydrocyclones (used in the majority of industrial circuits). Hydrocyclones tend to recycle a higher degree of fines, increasing the fines content of the product and thus the measured fractal dimension of breakage. If the inherent difference between the test and plant can be accounted for, the Bond test could be used to predict the fractal dimension of the industrial circuit.

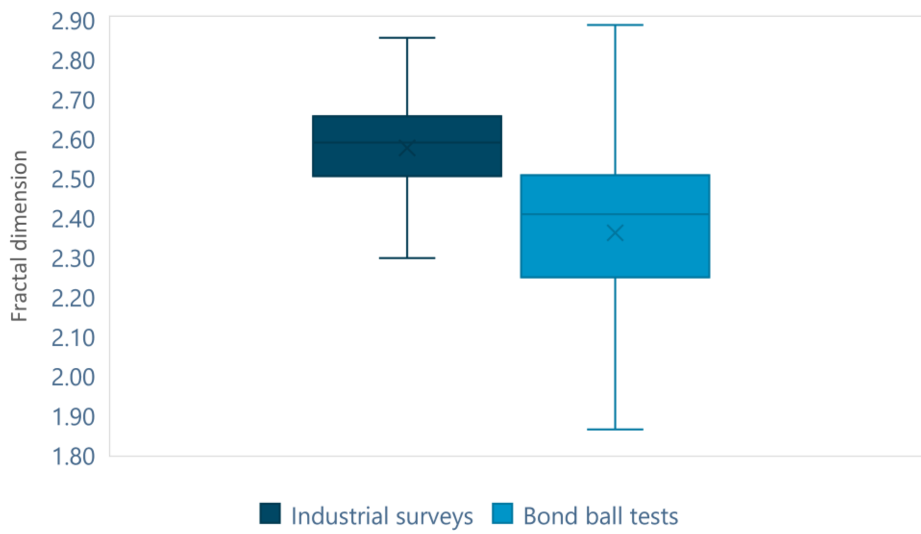

Figure 8 - Fractal dimensions recorded from the Bond ball mill work index test in relation to the fractal dimensions of industrial surveys

The distribution of size specific energy (SSE75) is presented in a similar way for both industrial surveys and the Bond ball mill tests. Figure 9 shows that the SSE75 was typically approximately 40% larger for the industrial circuits than the Bond tests. This difference is likely to be due to a combination of inefficiencies in the industrial circuits and that the fitting parameters uses in the Bond ball mill tests were developed for a ball mill in a standard circuit (staged crush, rod and ball mill) from the 1950s. The difference between the laboratory and industrial systems is well-known and engineering programs such as Ausgrind (Lane et. al., 2013) utilise efficiency factors to design various circuit configurations from standard ore characteristics.

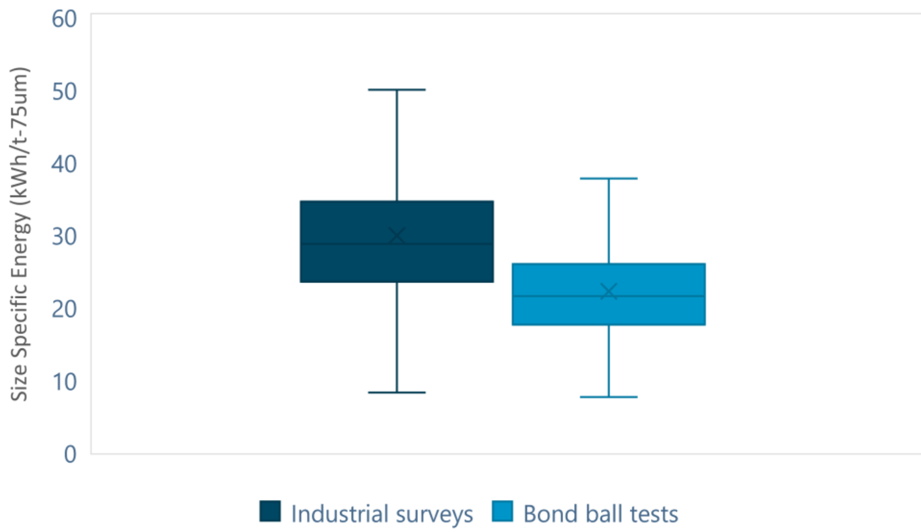

Figure 9 - Difference between SSE75 measured in the Bond ball mill test and measured in industrial surveys

6 Fractal Energy modelling

The fractal energy approach was used as an alternative methodology to corroborate the Ausgrind (Lane et. al., 2013) predictions for a design for a HPGR with edge recycle as the standard efficiency factors are not applicable to HPGR with edge recycle. The fractal energy approach allows the modelling of the full particle size distribution delivered by the HPGR with edge recycle to the following ball milling circuit. Typical staged-crush and ball milling circuit tend to have a fractal dimension of breakage close to 2.5, with feed and product sizes that tend to have a gradient of 0.5 in log/log space. HPGRs tend to produce more fines in the product relative to P80 and the average fractal dimension of breakage for the HPGR testwork with edge recycle was 2.7 (Figure 10).

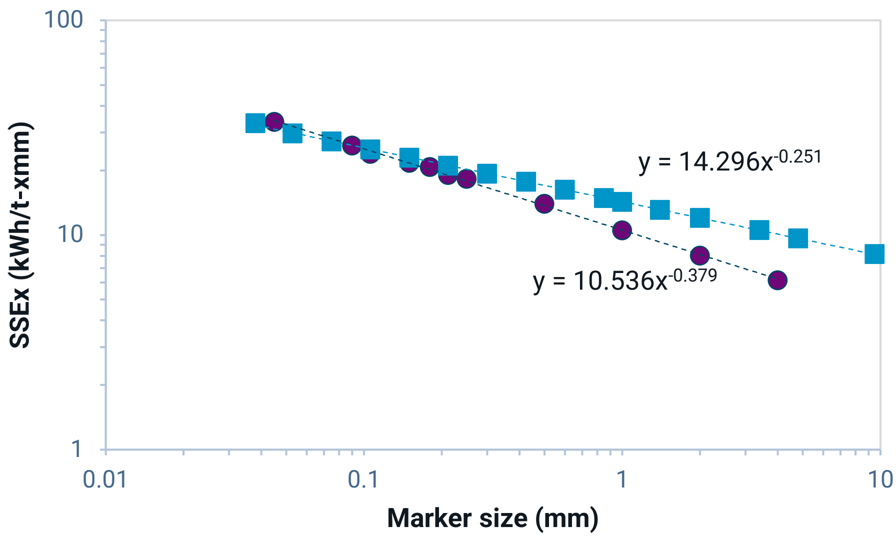

Figure 10 - Fractal dimension of breakage analysis on HPGR testwork with edge recycle.

Standard HPGRs operate with cheek plates on the side of the rolls. This results in an uneven pressure distribution across the rolls and coarser material in the HPGR product towards the edges of the rolls. Therefore, the edge recycle works as an effective size separation on the HPGR product. The proportion of HPGR product that reports to the edge recycle was analysed in this way for two industrial circuits. The edge recycle was modelled as an effective efficiency curve using the Whitten efficiency curve equation. The alpha parameter was similar for each tests (3-4), the D50c was related to the proceeding crusher screen aperture and the fines split was related to the proportion of edge recycle in the test. These efficiency curves were used to model the size distribution of the edge recycle, with crusher screen aperture and % edge recycle being used to determine the D50c and the fines split. Five iterations were used to determine the steady state recirculation of edge product back to the HPGR feed. The milling circuit was modelled using survey data as the basis. These parameters were used for modelling the milling circuit product from the HPGR circuit. The resultant particle size distributions are presented in Figure 11.

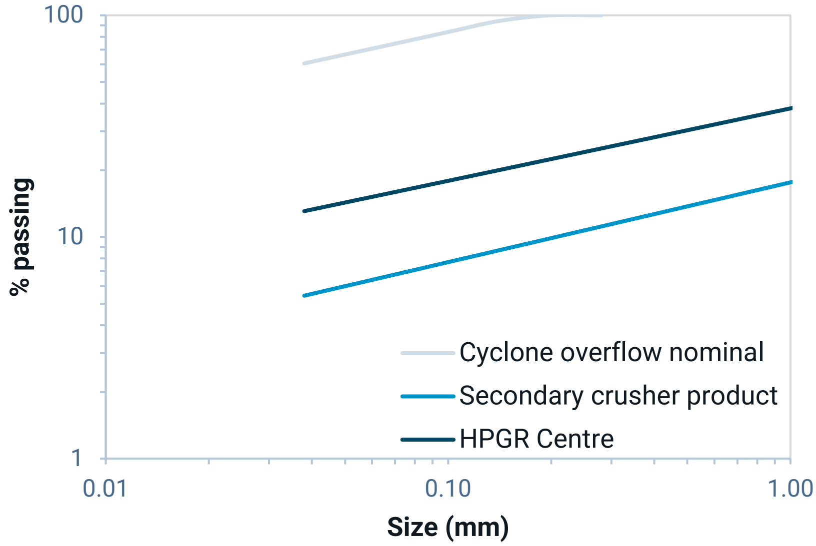

Figure 11 - Fractal Energy modelling output particle size distributions

7 Discussion

The Fractal Energy methodology combines the size specific energy (SSE) and fractal dimension of breakage to produce a comprehensive assessment of comminution performance. The SSE determines the energy required to generate new material finer than a single marker size. The fractal dimension of breakage is found by assessing SSE across a range of marker sizes. The combination of SSE and fractal dimension relates the energy input to the transformation of the full particle size distribution.

The Fractal Energy approach relates the energy input to the transformation of the size distribution from feed to product of the circuit. If both feed and product size distributions have the same gradient in log/log space, the fractal dimension of breakage is equal to 3 minus the gradient. However, if the feed size distribution has a gradient that is greater and the product size distribution, the fractal dimension will be equal to less than 3 minus the gradient of the product size distribution. And the opposite is also true. This phenomenon needs to be explored further with a mathematical proof. In the meantime, the trendline in Figure 12 approximates the relationship.

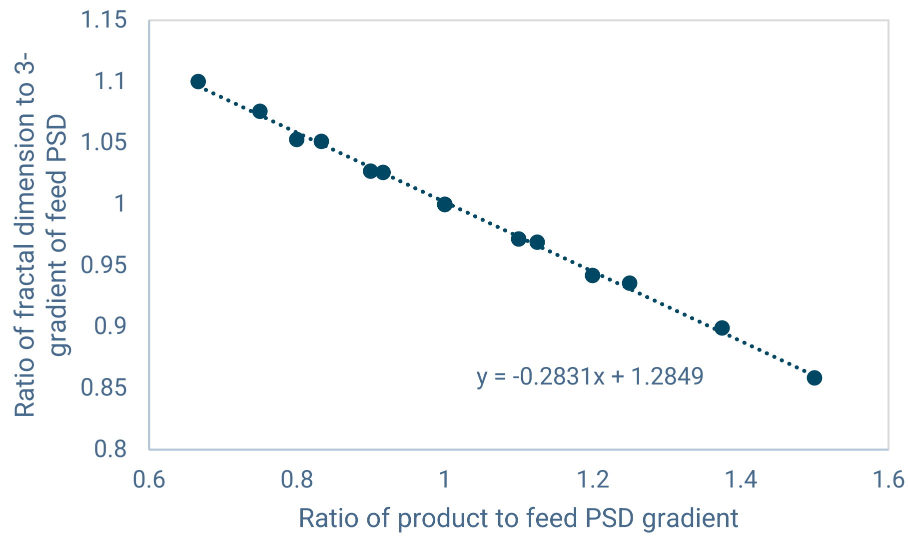

Figure 12 - Fundamental relationship between the gradient of the feed and product size distributions and the fractal dimension of breakage

The Fractal Energy approach also has the potential to unify the various comminution theories (Bond, Morrell, Rittinger, Kick, Hukki). The Fractal Energy approach provides a general theory of which the previous theories encompass specific cases (see Figure 13). For instance, the Bond equation is found when the fractal dimension equation equals 2.5 and Morrell is found when the fractal dimension equals (3 - 0.295 + P_{80}/1000000).

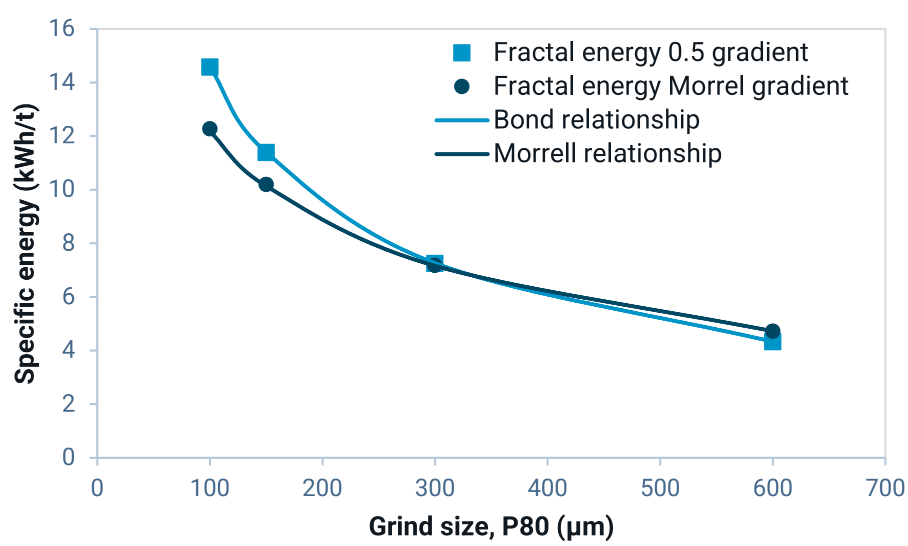

Figure 13 - Example to show that the Morrell and Bond relationship are specific examples of the general Fractal Energy approach

The competence and mineralogy of different ore-types impact the SSE and fractal dimension measured. However, the difference in the Fractal Energy approach between SAG and ball mills on the same ore is minor because the breakage mechanism for the fines are similar. There is evidence that different breakage mechanisms achieved in stirred mills and HPGRs produce different results, which needs to be explored further.

8 Conclusions

The Fractal Energy approach has been presented for different ore-types, comminution equipment and laboratory ore characterisation tests. The headline conclusions are:

1. The Fractal Energy approach allows the prediction of a full product size distribution, in a theoretically sound approach, without having to employ the complicated population balance method.
2. The efficiency of fines production with the application of energy is determined by the SSE term.
3. The transformation of the gradient of the feed size distribution into the product size distribution is determined through the fractal dimension.
4. The Fractal Energy approach has been shown to be consistent when the ore and breakage environment are similar, and responds logically for different ore-types, equipment and circuit configurations.
5. Fractal Energy modelling can be used to determine the size distributions around comminution circuits. This approach was followed to estimate the size distributions around a HPGR, ball milling circuit with edge recycle. The results show that this approach provides a realistic estimate of the performance of non-standard circuit configurations.

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